Statistics of Self-quenching Time in Single Photon Avalanche Diodes

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Single-photon avalanche diodes (SPADs) convert a single photo-excitation event, resulting from the absorption of a photon, into a measurable self-sustaining current in the external circuit consisting of a DC-bias source and a series load resistor. This avalanche current is produced with a certain probability that depends upon the bias voltage and the SPAD's structure. The mechanism for generating the self-sustaining avalanche current is the cascade of impact ionizations in the multiplication region of the SPAD, which occurs at or beyond the condition of avalanche breakdown. The breakdown condition corresponds to the smallest electric field (or bias) at which the multiplication factor of an avalanche photodiode becomes infinite, on average; equivalently, it is the smallest electric field at which the breakdown probability is nonzero. In practice, a SPAD is biased slightly above the breakdown voltage to maximize the probability that avalanche breakdown occurs without introducing too many dark carriers (that may result from band-to-band tunneling, for example) that can result in false counts.

Upon triggering avalanche breakdown, current through the diode and the load resistor can rise quickly until the excess bias (above breakdown) is built up across the resistor, leaving the SPAD biased precisely at breakdown [1] while the current fluctuates around the mean value required to maintain this bias. The avalanche current eventually collapses via such a statistical fluctuation, thereby quenching the avalanche current until another trigger occurs. The time measured from the onset of the self-sustaining avalanche current to its self-termination is referred to as the SPAD's *quenching time*. Evidently, this time is stochastic and it depends upon the load resistance as well as the avalanche, resistive and capacitive properties of the SPAD itself [1].

The purpose of this summary is to report our recent probabilistic modeling of the statistics of the quenching time. Understanding the time-scale of the quenching time as well as its fluctuations allows us to predict the statistics of the amount of charge passing through the diode due to each photon detection, which, in turn, sheds light on the number of trapped carriers in the diode, and hence, on after-pulsing.

To calculate the statistics of the duration of the avalanche current, we observe that such current is generated by electrons and holes distributed throughout the multiplication region. We can regard these as primary carriers, each generating its own individual avalanche pulse, all of which flow in parallel to generate a mean avalanche current, *I*. For the avalanche current to self-quench, each of these individual avalanche pulses must terminate independently. The probability that this happens before time *t* elapses is given by $F_I(t) = \prod_i F_{e,h}(z_i, t)$, where the product is over all electrons and holes, situated at z_i in the multiplication region, and $F_{e,h}(z_i, t)$ is the probability that an electron (hole) injected at z_i will give rise to an avalanche pulse which terminates before time t has elapsed. We have shown theoretically that when the SPAD is biased precisely at

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breakdown (i.e., prior to the avalanche current collapsing), $F_{e,h}(z_i,t) \approx 1 - f_{e,h}(z_i)/t$. Interestingly, we note that this asymptotic behavior is different from those corresponding to below or above breakdown, for which the asymptotic behavior is exponential [2,3]. A representative example demonstrating this $1/t$ asymptotic phenomenon is shown in Fig. 1 (Left), where the $1/t$ asymptotic behavior is clearly evident. With this asymptotic result available and by utilizing the appropriate spatial distribution of electrons and holes at breakdown (which we have derived), it can be shown that $F_I(t) \approx \exp\left(-\frac{T}{t}\right)$, where $T = \frac{IC\tau_0^2 J}{q}$, $J = 2/\ln(k)\left(2/\ln(k) + \frac{1+k}{1-k}\right)$, q is the electronic charge, C is a dimensionless constant, τ_0 is the average electron and hole transit time across the multiplication region, and $k = \beta/\alpha$ is the hole-to-electron ionization-coefficient ratio. We observe that this probability distribution function has an infinite mean; however, its median is $t_{1/2} = T/ln(2)$.

We end by showing the dependence of the median quenching time on k, as shown in Fig. 1(Right), which suggests that SPADs with k values close to unity have a better quench-time performance than those with larger or smaller k values.

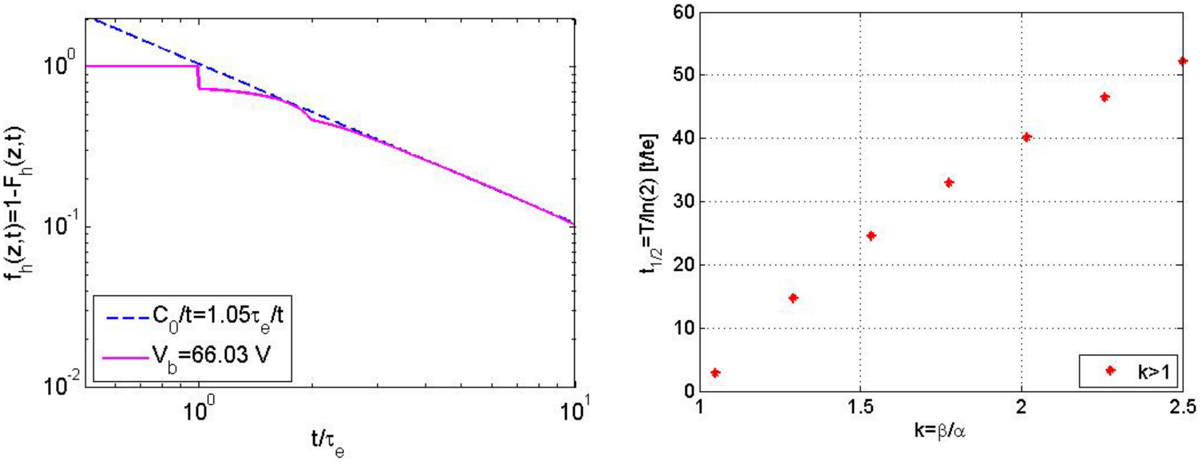

Fig. 1(Left): Calculated graph of one minus the probability distribution function of an avalanche pulse triggered by a parent electron at the edge (z=0) of the SPAD's multiplication region as a function of time normalized by the electron's transit time. This graph corresponds to a 1600-nm InP SPAD. **(Right):** Calculated median of the quenching time as a function of the ionization-coefficient ratio k.

In conclusion, we have developed an analytical probabilistic model to calculate the probability distribution function of quenching time for passive-quenching SPAD circuits. This model can be used to optimize the design of SPADs for specific applications.

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